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FAQCTUARY 2022-1

HOW TO ASSESS BROKER'S PERFORMANCES IN AN ELEMENTARY WAY?

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February 2022

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ABSTRACT

In order to rate intermediaries, Detralytics has developed a credibility model separating pure randomness from structural differences in loss ratios. The output results in a reliability index quantifying the confidence one can place on underwriting experience for a particular intermediary (based on the credibility coefficient) and a performance index assessing the quality of the intermediary's production (based on the credibility predictor of individual random effects). In this note, an elementary approach is proposed, treating the broker's efficiency level as a fixed effect (rather than a random effect in credibility). This provides the actuary with a preliminary ranking pointing to the ineffective intermediaries.

Contents

1 Context

Consider B brokers. For each broker b, $b = 1, \ldots, B$, we denote by $S_{b1}, \ldots, S_{bn_b}$ and $P_{b1}, \ldots, P_{bn_b}$ the annual claim amounts and technical premiums, respectively, where n_b denotes the number of years of observation. The corresponding loss ratios are given by $Y_{bj} = \frac{S_{bj}}{P_{bj}}$, $b = 1, \ldots, B$ and $j = 1, \ldots, n_b$.

2 Assumptions

We make the following assumptions:

(A1) The loss ratio Y_{bj} is distributed as follows:

$$Y_{bj} \sim Nor\left(\theta_b, \frac{\sigma^2}{P_{bj}}\right). \tag{2.1}$$

(A2) For a given broker b, the loss ratios $Y_{b1}, \ldots, Y_{bn_b}$ are independent.

(A3) The loss ratios of 2 different brokers are independent, that is, $(Y_{b1}, \ldots, Y_{bn_b})$ and $(Y_{b'1}, \ldots, Y_{b'n_{b'}})$ are independent for $b \neq b'$.

The assumption A1 is reasonable when the brokers under consideration have sufficiently large business volume. Notice that the Normal distribution can be replaced with the Student's t-distribution in case the loss ratios have heavier tails.

3 Estimation

Let $w_{bj} = P_{bj}$, so that (2.1) can be rewritten as

$$Y_{bj} \sim Nor\left(\theta_b, \frac{\sigma^2}{w_{bj}}\right).$$

We recognize a standard linear regression so that using least squares (or maximum likelihood estimation) gives the following estimates for θ_b and σ^2 :

$$\widehat{\theta}_b = \frac{\sum_{j=1}^{n_b} w_{bj} Y_{bj}}{\sum_{j=1}^{n_b} w_{bj}}, \quad b = 1, \ldots, B, \tag{3.1}$$

$$\widehat{\sigma}^2 = \frac{1}{N - B} \sum_{b=1}^{B} \sum_{j=1}^{n_b} w_{bj} \left(Y_{bj} - \widehat{\theta}_b\right)^2, \tag{3.2}$$

where $N = \sum_{b=1}^{B} n_b$ is the total number of observations.

4 Confidence intervals

From assumptions A1 and A2, the expression (3.1) for $\widehat{\theta}_b$ directly leads to

$$\widehat{\theta}_b \sim Nor\left(\theta_b, \frac{\sigma^2}{\sum_{j=1}^{n_b} w_{bj}}\right). \tag{4.1}$$

Hence, for $\alpha \in (0,1)$, a $(1-\alpha)$ confidence interval can be established for θ_b, that is,

$$CI_{1-\alpha}^{\theta_b} = \left[\widehat{\theta}_b - z_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}, \widehat{\theta}_b + z_{\alpha/2}\frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}\right], \tag{4.2}$$

where $z_{\alpha/2}$ is the percentile at a confidence level $1-\alpha/2$ for the standard Normal distribution. A typical value for α is 5%, which gives

$$CI_{95\%}^{\theta_b} = \left[\widehat{\theta}_b - 1.96\frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}, \widehat{\theta}_b + 1.96\frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}\right]. \tag{4.3}$$

5 Statistical test

Consider the null hypothesis $H_0 : \theta_b \leq 1$. This hypothesis means that broker b attracts policyholders that are less risky than their risk classes. In a word, broker b can then be seen as a good business provider. The null hypothesis will be rejected at the level $\alpha \in (0,1)$ when

$$\widehat{\theta}_b > 1 + z_\alpha \frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}, \tag{5.1}$$

where z_α is the percentile at a confidence level $1-\alpha$ for the standard Normal distribution. Selecting $\alpha = 5\%$, the null will be rejected when

$$\widehat{\theta}_b > 1 + 1.645\frac{\widehat{\sigma}}{\sqrt{\sum_{j=1}^{n_b} w_{bj}}}. \tag{5.2}$$

Of course, the message given here is very simple. On the one hand, the model presented here should be used as a first approach and contains many limitations. On the other hand, in practice, further analysis should be conducted in order to understand the reasons why some brokers appear to be less efficient than others.

6 Numerical example

We consider the following parameters:

- $B = 100$: there are 100 brokers;
- $n_b = 5$ for every $b = 1, \ldots, B$: every broker has been observed during 5 years;

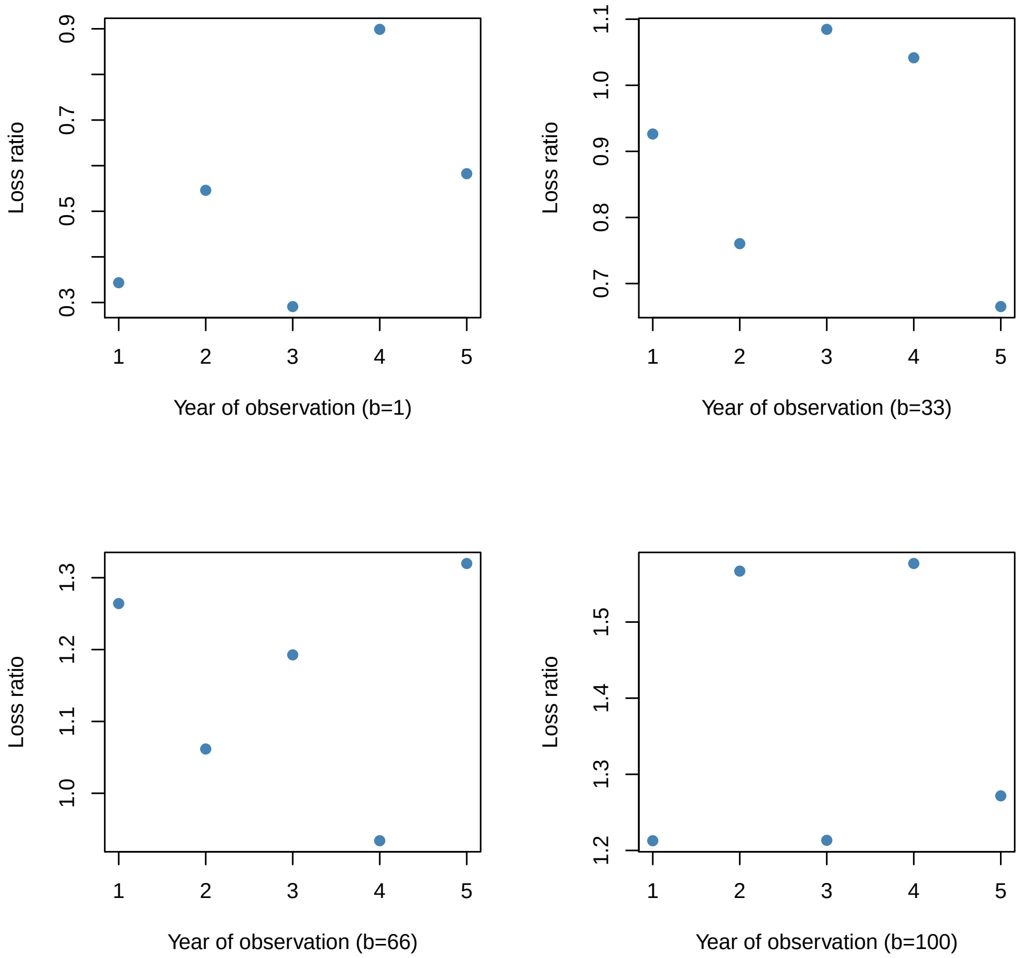

Figure 1: Simulated loss ratios for brokers $b = 1$, $b = 33$, $b = 66$ and $b = 100$.

- $w_{bj} = 1$ for every $b = 1, \ldots, B$ and $j = 1, \ldots, n_b$;
- $\sigma = 0.25$;
- $\theta_b = 0.5 + (b - 1) \times 0.01$: the brokers are increasingly inefficient (from $\theta_1 = 0.5$ for the first broker to $\theta_B = 1.49$ for the last one).

For each broker b, we simulate the corresponding loss ratios Y_{bj}. Figure 1 shows the simulated loss ratios for 4 brokers.

We get $\widehat{\sigma} = 0.254677$. In particular, inequality (5.2) becomes

$$\widehat{\theta}_b > 1 + 1.645 \times \frac{0.254677}{\sqrt{5}} = 1.187357. \tag{6.1}$$

In Figure 2, the black points represent the true values θ_b, the other points the estimates $\widehat{\theta}_b$ and the gray horizontal bars the corresponding confidence intervals at 95%. The vertical red

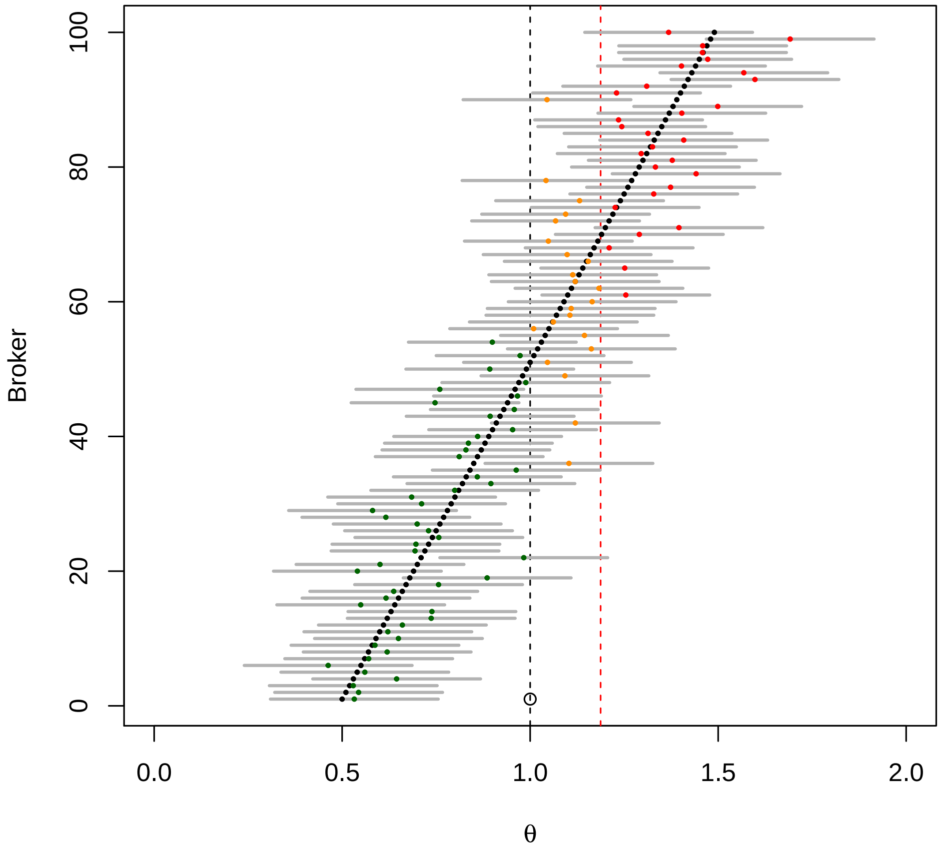

Figure 2: Black points: θ_b, green points: $\widehat{\theta}_b \leq 1$, orange points: $1 < \widehat{\theta}_b \leq 1.187357$, red points: $\widehat{\theta}_b > 1.187357$, horizontal bars: confidence intervals at 95%, vertical red dotted line: the threshold 1.187357 in (6.1).

dotted line corresponds to the lower bound 1.187357 for $\widehat{\theta}_b$ that is obtained in (6.1). The red points represent the brokers for which the null hypothesis $\theta_b \leq 1$ is rejected while the orange points are the brokers for which $1 < \widehat{\theta}_b \leq 1.187357$, i.e. the brokers that seem to be less efficient but for which one cannot reject the fact that they are actually efficient. Finally, the blue points are the brokers with $\widehat{\theta}_b \leq 1$. It is interesting to notice that the brokers for which we reject H_0 are indeed brokers with $\theta_b > 1$.

In Figure 3, we depict the results for $n_b = 10$, i.e. when we double the number of years of observations. As expected, the confidence intervals are smaller than when $n_b = 5$.

Finally, in Figure 4, we depict the results for $\sigma = 0.5$, i.e. when we double the standard deviations of the loss ratios. Of course, since we increase the volatility of the loss ratios, we increase the uncertainty around the estimates.

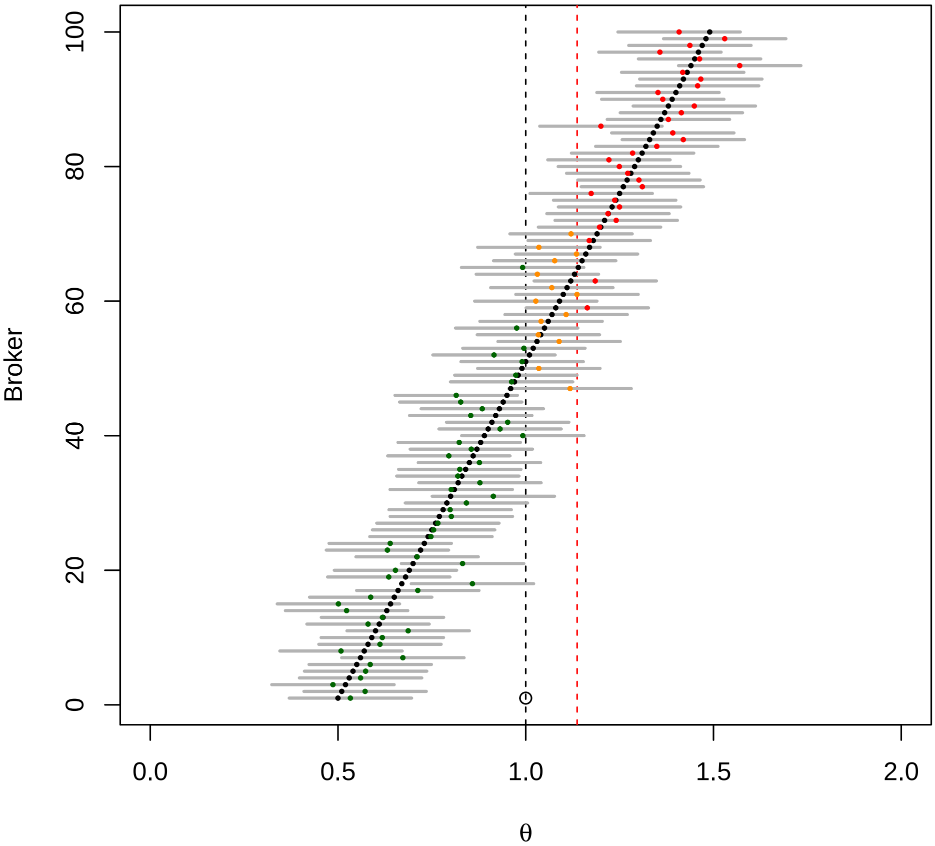

Figure 3: Results for $n_b = 10$.

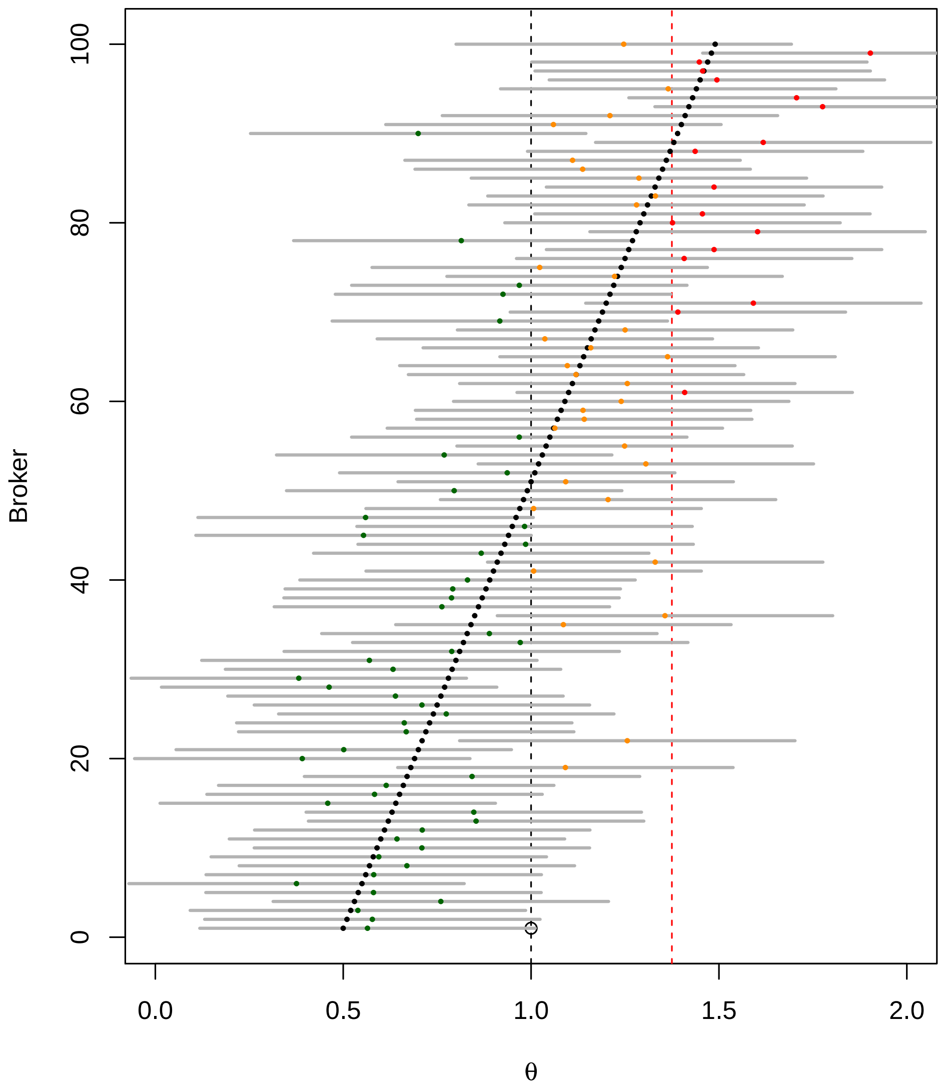

Figure 4: Results for $\sigma = 0.5$.

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